ELSEVIER

Journal of Fluids and Structures 25 (2009) 1369–1370

JOURNAL OF FLUIDS AND STRUCTURES

www.elsevier.com/locate/jfs

Special Brief Communication

The formula for motion magnitudes of cylinders experiencing vortex-shedding

Li Lee*, Don Allen

Shell Global Solutions (US) Inc., P.O. Box 4327, Houston, TX 77210, USA

Received 21 July 2009; accepted 28 August 2009
Available online 24 October 2009

Abstract

This brief communication questions the theoretical basis of the Griffin plot for VIV response prediction and suggests to use the original formula for this purpose.

Keywords: Vortex-induced vibration (VIV); Motion magnitudes; Griffin plot

While reviewing test data for designing subsea pipeline free spans, the authors found the following: when experiencing vortex-shedding, the first mode in-line motion magnitudes of two equal-diameter and length, flexible cylinders of different bending stiffness, were significantly different: the stiffer structure has smaller responses. This discovery prompted the authors to re-examine the fundamental equations governing the motion, as detailed below.

The nth modal equation of motion of a flexible beam can be written as

$$M_n \mathrm{d}^2 q_n(t)/\mathrm{d}t^2 + 2\zeta_n M_n \omega_n \mathrm{d}q_n(t)/\mathrm{d}t + K_n q_n(t) = Q_n(t), \quad (1)$$

where $q_n(t)$ is the modal displacement, M_n the modal mass, $Q_n(t)$ the modal force, K_n the modal stiffness, and ζ_n the modal damping. The modal force is given by $Q_n(t) = Q_n \sin(\varpi t)$, with ϖ the excitation frequency and Q_n the force amplitude. The modal displacement for the nth mode is

$$q_n(t) = q_n \sin(\varpi t - \theta_n), \quad (2)$$

where θ_n is a phase angle, given by $\tan^{-1}[2\zeta_n\beta_n/(1-(\beta_n)^2)]$, and β_n the ratio of the excitation frequency (ϖ) to the modal natural frequency (ω_n). The modal displacement amplitude, q_n, can be expressed as

$$q_n = (Q_n/K_n)(1/[(1-(\beta_n)^2)^2 + (2\zeta_n\beta_n)^2]^{1/2}). \quad (3)$$

The physical displacement, $y(x, t)$, can be linked to the modal displacement through mode shape $\Phi_n(x)$:

$$y(x,t) = \sum_n \Phi_n(x) q_n(t). \quad (4)$$

The modal mass, stiffness and frequency are related via

$$\omega_n^2 = K_n/M_n. \quad (5)$$

*Corresponding author.
E-mail address: li.lee@shell.com (L. Lee).

0889-9746/$ - see front matter
doi:10.1016/j.jfluidstructs.2009.08.004

For a pinned–pinned beam, assuming that the flow is uniform and that the vortices are fully correlated along its entire length exposed to the flow, the magnitude of the modal force, Q_n, is

$$Q_n = (2L/\pi)(1/2)\rho D C_{do} V^2, \tag{6}$$

where ρ is the fluid density, D the cylinder outside diameter, C_{do} the oscillating force coefficient, L the cylinder length, and V the flow speed. Note that the assumption on the force distribution only simplifies the expression and it has no effect on the conclusions.

Once the structure vibrates at its natural frequency ($\beta_n = 1.0$), q_n becomes

$$q_n = (Q_n/K_n)(1/2\zeta_n) = Q_n/(\omega_n C_n) = (Q_n/C_n)\sqrt{(M_n/K_n)}, \tag{7}$$

where the modal damping constant $C_n = 2\zeta_n M_n \omega_n$. In the open literature such as in Blevins (1990), an equation for the motion amplitude, similar to Eq. (7), is further extended with a number of substitutions. In the following, the same procedure is used to demonstrate that if the substitutions are carried out, the structural (bending) stiffness will no longer be present.

The vortex-shedding frequency and the flow speed are linked by the Strouhal relation (strictly speaking, this is defined for a stationary cylinder only),

$$f_s = VS_t/D \quad \text{or} \quad \omega_s = 2\pi VS_t/D \tag{8}$$

where S_t is the Strouhal number. Incorporating Eqs. (5), (6) and (8) into (7) with $n = 1$ (for the first mode motion), one obtains

$$q_1 = DC_{do}/\pi^2 S_t^2 K_s \quad \text{or conventionally,} \quad A/D = C_{do}/\pi^2 S_t^2 K_s, \tag{9}$$

where A is the motion amplitude and K_s the reduced damping given by

$$K_s = 4\pi m_e \zeta_1/\rho D^2. \tag{10}$$

Note that m_e is the mass per unit length of the beam, including added mass and that of the contents (if any). For a pinned–pinned beam with uniform mass properties, $M_1 = m_e L/2$. Eq. (9), often in a varied form, is the basis for the Griffin plot (Griffin and Ramberg, 1982) and VIV design guidelines such as in the Det Norske Veritas (2006) and ASME (1992) references. Comparing Eq. (9) with Eq. (7), one can see that the stiffness is no longer present. The essence of these substitutions is to equate the "rational" natural frequency formula to the definition of the vortex-shedding frequency, which, in our view, is questionable. Eq. (9) establishes, universally, reduced damping as the key parameter controlling the VIV motion magnitudes. The influence of the bending stiffness is no longer accounted for, and the impact of the mass may not be correctly considered.

We believe that the right formula for VIV motion magnitudes, once the motion has not significantly influenced vortex-shedding, should be the original Eq. (7). Further experiments on first mode in-line VIV motions have been conducted by the authors and their colleagues at Shell. A portion of the data, which correlate well with our theory, have been published at a recent technical conference (Lee et al., 2009). More definitive tests are being planned to reinforce our conclusions. It is also our wish that this communication generate enough interest so that other researchers in the VIV community could conduct their own experiments to investigate this important issue.

Acknowledgments

The authors would like to thank Shell for permission for this publication. They would also acknowledge Frans Kopp, Dean Henning, and Doug McMullen for their support.

References

ASME, 1992. Boiler and Pressure Vessel Code.

Blevins, R.D., 1990. Flow-Induced Vibration, second ed. Krieger Publishing Company.

Det Norske Veritas, 2006. DNV-RP-F105 Free Spanning Pipelines.

Griffin, O.M., Ramberg, S.E., 1982. Some recent studies of vortex shedding with application to marine tubulars and risers. ASME Journal of Energy Resources Technology 104, 2–13.

Lee, L., Allen, D.W., Pontaza, J.P., Kopp, F., Jhingran, V. 2009. In-line motion of subsea pipeline span models experiencing vortex-shedding, Offshore Mechanics and Arctic Engineering (OMAE) Conference Proceedings, no. 79084, Honolulu, USA.